An extension of Bernstein-Bézier surface over the triangular domain*

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Abstract In this paper, a set of quasi-Bernstein polynomials of degree n with one parameter is presented, which is an extension of the Bernstein polynomials over the triangular domain. Using the presented polynomials as basis functions, we construct a class of shape adjusting surfaces defined over the triangular domain with a shape parameter, namely, quasi-B-B parametric surfaces. These surfaces share many properties with the B-B parametric surfaces. In particular, when shape parameters equal 1, they degenerate to be the B-B parametric surfaces. By changing the value of the shape parameter, we can get different surfaces under the fixed control net.

Keywords: **triangular domain, shape parameter, Bézier surfaces.**

Bézier patches are one kind of the most wildly used surfaces in computer-aided geometric design (CAGD). It basically includes two categories in terms of different domain under consideration: tensor product patches over the rectangle domain and B-B parametric surfaces over the triangular domain. In practical applications, we usually need to adjust the shape and position of the Bézier patches. However, Bézier surface is determined by the Bézier basis functions and control net. Hence, after we adopt a set of Bézier basis functions to construct a surface, the shape of this surface can only be changed by adjusting the control net of it. Fortunately, the introduction of the rational Bézier surfaces can help us deal with surface modification without changing the control net. This larger class of Bézier surfaces provides more flexibility in surfaces design than the usual (non-rational) Bézier surfaces. This advantage of the rational Bézier surfaces derives from the introducing weights into the Bézier surfaces. But the calculations of derivative and integral are complicated due to the fractional expressions. It is also difficult to choose appropriate weight values to obtain the desired shape[1].

Recently, a curve shape modification by virtue of controlling shape parameters becomes an interesting topic and some progress has been made. The basic idea is that adding flexibility to the basis functions, namely, introducing a parameter into the basis functions. Then, the curves represented by these basis functions can have different shapes by varying shape parameter values. Several basis functions with a shape parameter have been constructed through different approaches. For example, in [2], polynomial blending functions of degree 3 were constructed by using polynomials of degree 4, then the redundant degree of freedom in the coefficients was used as a shape parameter. And C-Bézier curve was proposed by Chen and Wang[3]. In their paper, trigonometric functions were used as initial basis functions, then the general basis functions then could be obtained by a recurrence formula. The upper bound of the parametric interval of these trigonometric functions was actually a shape parameter. In terms of constructing basis functions by an integration approach, Wang and Wang proposed a series of splines such as uniform B-Spline/hyperbolic polynomial, trigonometric polynomial B-Spline and Bézier curve with a shape parameter[4—8] The curves constructed by these basis functions take different shapes with invariant control points by varying values of the shape parameters. By tensor product method, the surfaces with an adjustable shape over the rectangle domain become an easy extension of the curves with the adjustable shape mentioned above.

Nevertheless, the B-B parametric surface over the triangular domain is not a tensor product patch exactly. Hence, we cannot get the B-B parametric

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surface with an adjustable shape through the method of tensor product. B-B parametric surfaces are suitable for geometric modeling based on irregular and scattered data. By the use of surfaces constructed over non-degenerate triangular parameter domains, we can avoid degeneracy of rectangular patches, and triangular patches can also be used as triangular structure elements which have been widely used in the finite element analysis and can be constructed in arbitrary topology grids as well[9]. Although triangular Bézier surfaces are of importance in CAGD, we lack methods to construct triangular patches with an adjustable shape.

In this study, we attempted to construct a set of basis functions with a parameter over the triangular domain. Based on these basis functions, a kind of surfaces with an adjustable shape over the triangular domain was obtained. These surfaces share many properties with the B-B parametric surface over the triangular domain, such as affine invariance, convex hull property, corner point interpolation, boundary curves and corner point tangent plane. Owning to the shape parameter, we got one kind of more flexible surfaces under the fixed control net, and obtained triangular patches of different shapes by varying parameters values. In particular, these surfaces degenerate to be B-B parametric surfaces over the triangular domain when the parameter values are equal to 1.

1 Preliminary

Definition 1. Given a domain triangle T with vertices T_1, T_2, T_3, every point P in the interior of T can be expressed as

$$P = uT_1 + vT_2 + wT_3,$$

where the coefficients are determined by the areas of subtriangles of domain triangle

$$(u, v, w) = (\Delta PT_2T_3/\Delta T_1T_2T_3, \Delta T_1PT_3/\Delta T_1T_2T_3, \Delta T_1T_2P/\Delta T_1T_2T_3).$$

Here, $\triangle ABC$ denotes the area of the triangle with vertices A, B, C.

We call these coefficients the barycentric coordinates of P with respect to domain triangle $T_1T_2T_3$[10] (see Fig. 1(a)). Also we have the constraints $u, v, w \geqslant 0$, $u+v+w=1$.

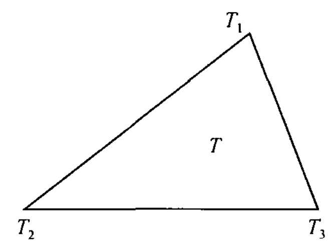
Definition 2. Given $(n+1)(n+2)/2$ vectors $\boldsymbol{T}_{i,j,k} \in \Re^3$, $i, j, k \geqslant 0$, $i+j+k=n$, and a domain triangle T and a point P in the interior of it with barycentric coordinates (u, v, w), we call

$$\boldsymbol{T}^n(u, v, w) = \sum_{i+j+k=n} B^n_{i,j,k}(u, v, w)\boldsymbol{T}_{i,j,k}, \quad (u, v, w) \in T, \quad u, v, w \geqslant 0, \quad u+v+w=1 \qquad (1)$$

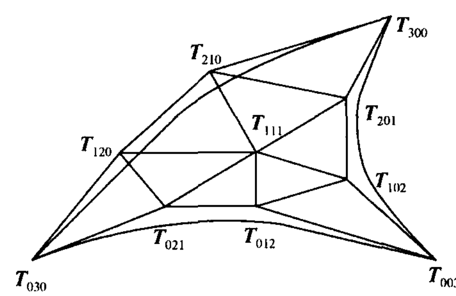
the Bernstein-Bézier parametric surface of degree n over the domain triangle T (abbreviated as B-B parametric surface), where

$$B^n_{i,j,k}(u, v, w) = \frac{n!}{i!j!k!}u^i v^j w^k,$$
$$i+j+k=n, \quad u, v, w \geqslant 0, \quad u+v+w=1$$

is called Bernstein polynomials of degree n, $\boldsymbol{T}_{i,j,k}$ are called the control points of surface (1); the surface which is composed of n^2 triangles $\boldsymbol{T}_{i+1,j,k}\boldsymbol{T}_{i,j+1,k}\boldsymbol{T}_{i,j,k+1}$ $(i+j+k=n-1)$ is referred to as the Bézier control net or B net of surface (1)(see Fig. 1(b)).



(a)



(b)

Fig. 1. Domain triangle and control net. (a) Domain triangle, T with vertices T_1, T_2, T_3; (b) control net with control points $\boldsymbol{T}_{i,j,k}$.

Bernstein polynomials have the properties of non-negativity, symmetry, normalization, roots, order of zeros, linear independence; and B-B parametric surfaces have the properties of affine invariance, convex hull property, corner point interpolation, boundary curve and corner point tangent plane.

2 Quasi-Bernstein polynomials with a parameter

Definition 3. The quasi-Bernstein polynomials of degree n with a parameter λ associated with a domain triangle T are defined as:

$$\widetilde{B}^n_{i,j,k}(u,v,w)=\begin{cases}\dfrac{n!}{i!j!k!}u^iv^jw^k\left[\left(\dfrac{n+1}{i+1}-\dfrac{j+k}{i+1}\lambda\right)u+\lambda(v+w)\right] & k=0, i>j+1 \\ & \text{or } i>j\geqslant k>0 \\ \dfrac{n!}{i!j!k!}u^iv^jw^k\left[\left(\dfrac{n+1}{i+1}-\dfrac{j+k}{i+1}\lambda\right)(u+v)+\left(\dfrac{n+1}{k+1}-\dfrac{i+j}{k+1}\lambda\right)w\right] & i=j>k+1.\end{cases}$$

In addition,

if n is odd and $k=0, i=j+1$

$$\widetilde{B}^n_{i,j,0}=\frac{n!}{i!j!}u^iv^j\cdot\left[\left(\frac{n+1}{i+1}-\frac{j}{i+1}\lambda\right)u+v+\lambda w\right];$$

if $n=3m-1,\ m\in N,\ i=j=k+1$

$$\widetilde{B}^n_{i,j,k}=\frac{n!}{i!j!k!}u^iv^jw^k\cdot\left[\left(\frac{2(n+1)}{n-k+2}-\frac{n+k}{n-k+2}\lambda\right)(u+v)+w\right];$$

if $n=3m,\ m\in N,\ i=j=k=m$

$$\widetilde{B}^n_{i,j,k}=\frac{n!}{m!m!m!}u^mv^mw^m\left(\frac{n+1}{m+1}-\frac{m}{m+1}\lambda\right)$$

with $u,v,w\geqslant 0,\ u+v+w=1,\ 0\leqslant\lambda\leqslant 1,\ i+j+k=n\,(n\geqslant 2)$. The subscripts of these polynomials satisfy $i\geqslant j\geqslant k$. If we display the subscripts of the polynomials in the triangular array, the subscripts satisfying $i\geqslant j\geqslant k$ will be located in the dashed area and its boundary (see Fig. 2(b)). The rest of the polynomials are defined symmetrically: when $j\geqslant i\geqslant k$, $\widetilde{B}^n_{i,j,k}(u,v,w)=\widetilde{B}^n_{j,i,k}(v,u,w)$; when $k\geqslant i\geqslant j$, $\widetilde{B}^n_{i,j,k}(u,v,w)=\widetilde{B}^n_{k,i,j}(w,u,v)$; when $k\geqslant j\geqslant i$, $\widetilde{B}^n_{i,j,k}(u,v,w)=\widetilde{B}^n_{k,j,i}(w,v,u)$; when $j\geqslant i\geqslant k$, $\widetilde{B}^n_{i,j,k}(u,v,w)=\widetilde{B}^n_{j,i,k}(v,u,w)$; when $j\geqslant k\geqslant i$, $\widetilde{B}^n_{i,j,k}(u,v,w)=\widetilde{B}^n_{j,k,i}(v,w,u)$.

These functions share many properties with the Bernstein polynomials over the triangular domain, such as:

Property 1 (Non-negativity). $\widetilde{B}^n_{i,j,k}\geqslant 0,\ i+j+k=n$.

Proof. This property can be simply derived form Definition 3.

Property 2 (Symmetry). Quasi-Bernstein polynomials are symmetric with respect to parameters u, v, w:

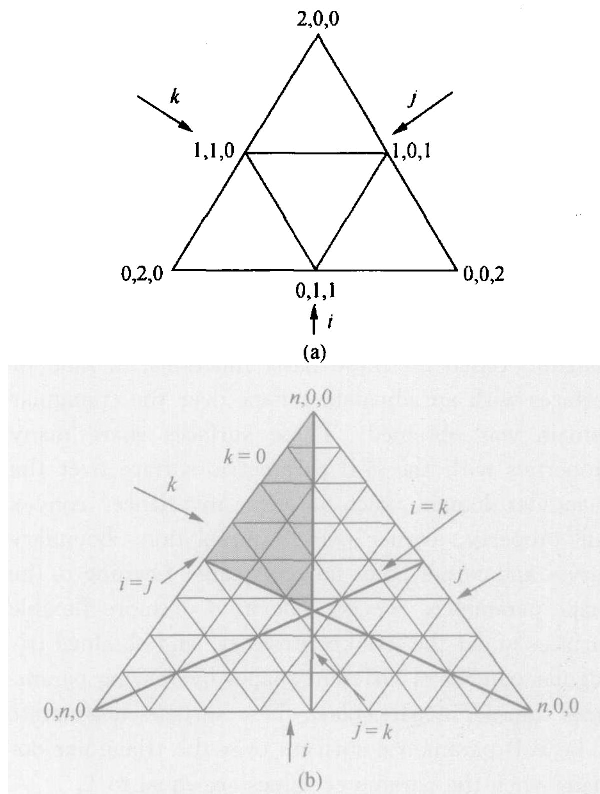


Fig. 2. Triangular array of the subscripts of quasi-Bernstein polynomials of degree n (the arrows denote the increasing direction of the variables beside them). (a) Triangular array of the subscripts when $n=2$; (b) the position of the subscripts which satisfy $i\geqslant j\geqslant k$ in the triangular array when n is even.

$$\begin{aligned}\widetilde{B}^n_{i,j,k}(u,v,w)&=\widetilde{B}^n_{j,i,k}(v,u,w)\\&=\widetilde{B}^n_{i,k,j}(u,w,v)=\widetilde{B}^n_{j,k,i}(v,w,u)\\&=\widetilde{B}^n_{k,i,j}(w,u,v)=\widetilde{B}^n_{k,j,i}(w,v,u)\end{aligned}$$

for $(u,v,w)\in T,\ u,v,w\geqslant 0,\ u+v+w=1,\ i+j+k=n$.

Proof. This property can be simply derived from Definition 3.

Property 3 (Normalization). $\sum_{i+j+k=n}\widetilde{B}^n_{i,j,k}=1$.

Proof. Applying Definition 3, and when $i\geqslant j\geqslant k$, in case $k=0$, we have if $i > j+1$,

$$\widetilde{B}^n_{i,j,k} = (1-\lambda)B^{n+1}_{i+1,j,k} + \lambda B^n_{i,j,k};$$

if $i = j+1$,

$$\widetilde{B}^n_{i,j,k} = (1-\lambda)\left(B^{n+1}_{i+1,j,k} + \frac{1}{2}B^{n+1}_{i,j+1,k}\right) + \lambda B^n_{i,j,k};$$

if $i = j$,

$$\widetilde{B}^n_{i,j,k} = (1-\lambda)(B^{n+1}_{i+1,j,k} + B^{n+1}_{i,j+1,k} + B^{n+1}_{i,j,k+1}) + \lambda B^n_{i,j,k}.$$

In case $k \neq 0$, we have

if $i > j \geqslant k$,

$$\widetilde{B}^n_{i,j,k} = (1-\lambda)B^{n+1}_{i+1,j,k} + \lambda B^n_{i,j,k};$$

if $i = j > k+1$ or $i = j = k$,

$$\widetilde{B}^n_{i,j,k} = (1-\lambda)(B^{n+1}_{i+1,j,k} + B^{n+1}_{i,j+1,k} + B^{n+1}_{i,j,k+1}) + \lambda B^n_{i,j,k};$$

if $i = j = k+1$,

$$\widetilde{B}^n_{i,j,k} = (1-\lambda)\left(B^{n+1}_{i+1,j,k} + B^{n+1}_{i,j+1,k} + \frac{1}{3}B^{n+1}_{i,j,k+1}\right) + \lambda B^n_{i,j,k}.$$

Denoting

$$A = \sum_{\substack{i,j\geqslant k\\ i+j+k=n}} \widetilde{B}^n_{i,j,k},$$

$$C = (1-\lambda)\sum_{\substack{I,J\geqslant K\\ I+J+K=n+1}} B^{n+1}_{I,J,K} + \lambda \sum_{\substack{i,j\geqslant k\\ i+j+k=n}} B^n_{i,j,k},$$

and following the symmetry, we obtain that when

$$n = 3m-1, \quad m \in N,$$

$$A = C - \frac{2}{3}(1-\lambda)B^{n+1}_{m,m,m}.$$

Similarly, when

$$n = 3m, \quad A = C + (1-\lambda)B^{n+1}_{m,m,m+1};$$

when

$$n = 3m+1, \quad A = C.$$

Following Property 2 and the property of normalization of the Bernstein polynomials, we have

$$\sum_{i+j+k=n} \widetilde{B}^n_{i,j,k} = (1-\lambda)\sum_{I+J+K=n+1} B^{n+1}_{I,J,K} + \lambda \sum_{i+j+k=n} B^n_{i,j,k} = 1-\lambda+\lambda = 1.$$

Property 4 (Degeneracy). When $\lambda = 1$, the quasi-Bernstein polynomials degenerate to be Bernstein polynomials over the triangular domain.

Property 5 (Roots). The quasi-Bernstein polynomials have roots in $[01]\times[01]\times[01]$:

$$\widetilde{B}^n_{i,j,k}(1,0,0) = \begin{cases} 1 & i = n \\ 0 & \text{other} \end{cases},$$

$$\widetilde{B}^n_{i,j,k}(0,1,0) = \begin{cases} 1 & j = n \\ 0 & \text{other} \end{cases},$$

$$\widetilde{B}^n_{i,j,k}(0,0,1) = \begin{cases} 1 & k = n \\ 0 & \text{other} \end{cases}.$$

Property 6 (Multiplicity of zeros).

In case $i+j+k = n$, if $v \neq 0$, $w \neq 0$:

if $\lambda \neq 0$, $(0, v, w)$ is i-fold zero of $\widetilde{B}^n_{i,j,k}$;

if $\lambda = 0$, $(0, v, w)$ is $i+1$-fold zero of $\widetilde{B}^n_{i,j,k}$($k = 0, i > j+1$; $j = 0$, $i > k+1$; $k \neq 0$, $i > j \geqslant k$; $j \neq 0$, $i > k \geqslant j$) and i-fold zero of the rest quasi-Bernstein polynomials.

In particular,

if $\lambda \neq 0$, $(0,0,1)$ is $(i+j)$-fold zero of $\widetilde{B}^n_{i,j,k}$;

if $\lambda = 0$, $(0,0,1)$ is $(i+j+1)$-fold zero of $\widetilde{B}^n_{i,j,k}$($i, j \geqslant k$, $i \neq j$; $j = 0, i \geqslant k+1$; $j \neq 0$, $i > k \geqslant j$; $i = 0, j > k+1$; $i \neq 0, j > k \geqslant i$), and $(i+j)$-fold zero of the rest quasi-Bernstein polynomials.

We can obtain the multiplicity of zero $(u,0,w)$ $(u \neq 0, w \neq 0)$, $(u,v,0)$ $(u \neq 0, v \neq 0)$, $(1,0,0)$, $(0,1,0)$ of the quasi-Bernstein polynomials respectively by applying the property of symmetry similarly.

Property 7 (Linear independence).

$$\sum_{i+j+k=n} a_{i,j,k}\widetilde{B}^n_{i,j,k} = 0 \quad \text{iff} \quad a_{i,j,k} = 0, \quad i+j+k = n.$$

Proof. The sufficiency is obvious. We prove the necessity as follows: If $\sum_{i+j+k=n} a_{i,j,k}\widetilde{B}^n_{i,j,k} = 0$, $a_{i,j,k} \in R$, $i+j+k = n$, like the proof of the property of normalization, we can rewrite the quasi-Bernstein polynomials as the combination of Bernstein polynomials of degree n and $n+1$, hence we have

$$\sum_{i+j+k=n} a_{i,j,k}\widetilde{B}^n_{i,j,k} = (1-\lambda)\sum_{I+J+K=n} T^{n+1}_{I,J,K}B^{n+1}_{I,J,K} + \lambda \sum_{i+j+k=n} a_{i,j,k}B^n_{i,j,k} = 0,$$

where

$$T_{I,J,K}^{n+1} = \sum_{i+j+k=n} s_{I,J,K}^{i,j,k} a_{i,j,k},$$
$$s_{I,J,K}^{i,j,k} \in \left\{0, 1, \frac{1}{2}, \frac{1}{3}\right\}.$$

Because $1-\lambda, \lambda$ are linearly independent, we have $\sum_{i+j+k=n} B_{i,j,k}^{n} a_{i,j,k} = 0$. Applying the property of linear independence of the Bernstein polynomials, we have $a_{i,j,k} = 0$, $i+j+k=n$, hence, the quasi-Bernstein polynomials are linearly independent, which form a basis.

3 Quasi-B-B parametric surface

Definition 4. Given $(n+1)(n+2)/2$ vectors $\boldsymbol{T}_{i,j,k} \in \Re^3$, $i,j,k \geqslant 0$, $i+j+k=n$, a domain triangle T and a point P in the interior of it with barycentric coordinate (u, v, w), we call

$$\widetilde{\boldsymbol{T}}^n(u,v,w) = \sum_{i+j+k=n} \widetilde{B}_{i,j,k}^{n}(u,v,w)\boldsymbol{T}_{i,j,k},$$
$$(u,v,w) \in T, \quad u,v,w \geqslant 0,$$
$$u+v+w=1$$

the quasi-Bernstein-Bézier parameter surfaces of degree n over the domain triangle T with a shape parameter λ, and abbreviate it as quasi-B-B parametric surfaces. These surfaces have the following properties:

Property 8 (Affine invariant and geometric invariant).

The shape of the surface is only dependent of the control points and independent of the underlying coordinate system. If the surface is transformed by an affine transformation, we could just as well apply the transformation to the control net and would end up with the same surface.

Proof. Because the surface is an affine combination of the control points, it is invariant under affine maps[9,11].

Property 9 (Convex hull property). The surface is in the convex hull of the control net.

Proof. This follows from properties of the nonnegativity and normalization of the quasi-Bernstein polynomials.

Property 10 (Corner point interpolation). The three corner points of the surface interpolate the corresponding corner points of the control net respectively.

Proof. Applying Property 5, we have:

$$\widetilde{\boldsymbol{T}}^n(1,0,0) = \sum_{i+j+k=n} \widetilde{B}_{i,j,k}^{n}(1,0,0)\boldsymbol{T}_{i,j,k}$$
$$= \widetilde{B}_{n,0,0}^{n}(1,0,0)\boldsymbol{T}_{n,0,0} = \boldsymbol{T}_{n,0,0}.$$

Similarly

$$\widetilde{\boldsymbol{T}}^n(0,1,0) = \boldsymbol{T}_{0,n,0}, \quad \widetilde{\boldsymbol{T}}^n(0,0,1) = \boldsymbol{T}_{0,0,n},$$

which completes the proof.

Property 11 (Corner point tangent plane). The points

$\{\boldsymbol{T}_{n,0,0}, \boldsymbol{T}_{n-1,1,0}, \boldsymbol{T}_{n-1,0,1}\}$ determine the tangent plane at $\boldsymbol{T}_{n,0,0}$,

$\{\boldsymbol{T}_{0,n,0}, \boldsymbol{T}_{1,n-1,0}, \boldsymbol{T}_{0,n-1,1}\}$ determine the tangent plane at $\boldsymbol{T}_{0,n,0}$,

$\{\boldsymbol{T}_{0,0,n}, \boldsymbol{T}_{1,0,n-1}, \boldsymbol{T}_{0,1,n-1}\}$ determine the tangent plane at $\boldsymbol{T}_{0,0,n}$.

Proof. We obtain the tangent plane at $\boldsymbol{T}_{n,0,0}$ firstly.

Recall that

$$\widetilde{\boldsymbol{T}}^n(u,v,w) = \widetilde{\boldsymbol{T}}^n(u,v,1-u-v),$$

from Property 6, we have

$$\left.\frac{\partial \widetilde{\boldsymbol{T}}^n(u,v,w)}{\partial u}\right|_{(1,0,0)} = \left.\frac{\partial \widetilde{B}_{n,0,0}^{n}(u,v,1-u-v)}{\partial u}\right|_{(1,0,0)} \boldsymbol{T}_{n,0,0} + \left.\frac{\partial \widetilde{B}_{n-1,0,1}^{n}(u,v,1-u-v)}{\partial u}\right|_{(1,0,0)} \boldsymbol{T}_{n-1,0,1}$$
$$= (n+1-\lambda)(\boldsymbol{T}_{n,0,0} - \boldsymbol{T}_{n-1,0,1}),$$

$$\left.\frac{\partial \widetilde{\boldsymbol{T}}^n(u,v,w)}{\partial v}\right|_{(1,0,0)} = (n+1-\lambda)(\boldsymbol{T}_{n-1,1,0} - \boldsymbol{T}_{n-1,0,1}).$$

It means that the tangent plane at $\boldsymbol{T}_{n,0,0}$ is just the plane through the three points $\boldsymbol{T}_{n,0,0}, \boldsymbol{T}_{n-1,0,0}, \boldsymbol{T}_{n-1,0,1}$. We can get the tangent plane at the rest of the corner points similarly.

Property 12 (Boundary curves). Boundary curves are Bézier curves with an adjustable shape.

Proof. We prove that when $w=0$, the surface reduces to a Bézier curve with a shape parameter.

From Definition 4, we have:

If $w=0$, $k \neq 0$ then $v = 1-u$, $\widetilde{B}_{i,j,k}^{n}(u,v,0) = 0$,

$$\widetilde{T}^n(u,v,0) = \sum_{i+j=n} \widetilde{B}^n_{i,j,0}(u,v,0)T_{i,j,0}$$
$$= \sum_{I+J=n+1} \frac{(n+1)!}{I!J!} u^I (1-u)^J \overline{T}^{n+1}_{I,J}$$
$$+ \sum_{i+j=n} \frac{n!}{i!j!} u^i (1-u)^j \overline{T}^n_{i,j}, \quad (2)$$

where

$$\overline{T}^{n+1}_{I,J} = (1-\lambda) \sum_{i+j+k=n} t^{i,j}_{I,J} T_{i,j,0},$$
$$t^{i,j,k}_{I,J} \in \left\{0, 1, \frac{1}{2}\right\},$$
$$\overline{T}^n_{i,j} = \lambda T_{i,j,0}, \quad i+j=n.$$

The first item of the right part of (2) is actually a Bézier curve of degree $n+1$ with control points $\overline{T}^{n+1}_{I,J}$. The second item is also a Bézier curve of degree n with control points $\overline{T}^n_{i,j}$, which can be represented as a Bézier curve of degree $n+1$ after degree arising. By collecting it to the first item, we get a Bézier curve of degree $n+1$ with a parameter λ. The other boundary curves can be obtained analogously.

4 An example

An example of a surface of degree 3 is given in Fig. 3. Fig. 3(a) shows the figure of the surface when the parameter's value is equal to 1 and Fig. 3(b) shows the figure when λ is equal to 0. The figures indicate that we can get surfaces of different shape by choosing different values of the shape parameter under the fixed control net.

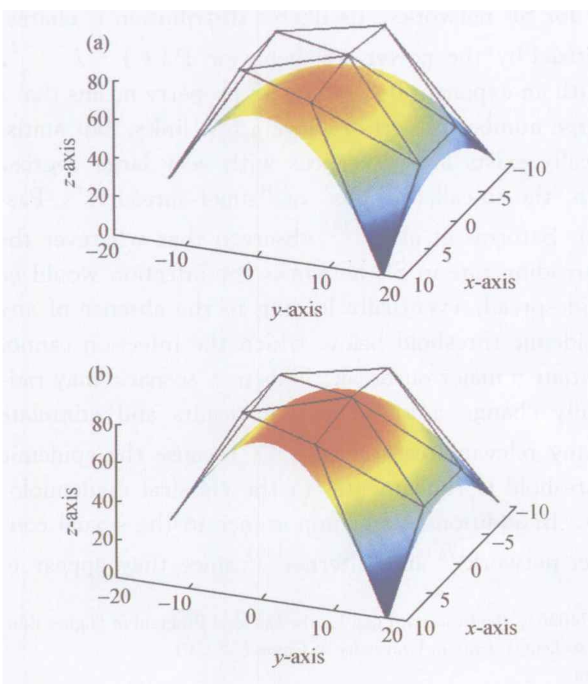


Fig. 3. The test surface when λ is equal to 1 (a) and 0 (b) respectively.

5 Conclusion

Here we propose a set of polynomial functions defined over the triangular domain with a parameter λ. They degenerate to be Bernstein polynomials defined over the triangular domain when λ is equal to 1. These functions have some excellent properties similar to the Bernstein polynomials, such as non-negativity, symmetry, normalization, linear independence, roots, multiplicity of zeros and so on. Based on this set of basis functions, we have constructed a kind of surfaces with an adjustable shape under a fixed control net. These surfaces share many properties with the B-B parametric surface, such as affine invariance, convex hull property, corner-point interpolation, boundary curves and corner point tangent plane. Particularly, they degenerate be to B-B parametric surfaces over the triangular domain when λ is equal to1. Since the important role of triangular patches in the shape modeling and the lack of the method of constructing surfaces with an adjustable shape over the triangular domain through adjusting shape parameters, the approach we proposed will make some contribution to practical applications.

References

1 Mainar E., Pena J. M. and Sánchez-Reyes J. Shape preserving alternatives to the rational Bézier model. Computer Aided Geometric Design, 2001, 18(1): 37—60.

2 Han X. L. and Liu S. J. An extension of the cubic uniform B-Spline curves. Journal of Computer-Aided Design & Computer Graphics, 2003, 15(5): 576—578.

3 Chen Q. Y. and Wang G. Z. A class of Bézier-like Curves. Computer Aided Geometirc Design, 2003, 20(1): 29—39.

4 Wang W. T. and Wang G. Z. Trigonometric polynomial B-Spline with shape parameter. Progress in Natural Science, 2004, 14 (11): 1023—1026.

5 Wang W. T. and Wang G. Z. Uniform B-Spline with shape parameter. Journal of Computer-Aided Design & Computer Graphics (in Chinese), 2004, 16(6): 783—788.

6 Wang W. T. and Wang G. Z. Trigonometirc polynomial uniform B-Spline with shape parameter. Chinese Journal of Computer (in Chinese), 2005, 28(7): 1192—1198.

7 Wang W. T. and Wang G. Z. Hyperbolic polynomial uniform B-Spline with shape parameter. Journal of Software (in Chinese), 2005, 16(4): 625—633.

8 Wang W. T. and Wang G. Z. Bézier Curves with shape parameter. Journal of Zhejiang University Science, 2005, 6A(6): 497—501.

9 Shi F. Z. Computer Aided Geometric Design & Non-Uniform Rational B-Spline. 1st ed. Bejing: Higher Education Press (in Chinese), 2001, 474—479.

10 Farin G. E. Triangular Bernstein-Bézier patches. Computer Aided Geometric Design, 1986, 3(2): 83—127.

11 Farin G. E. Curvature continuity and offsets for piecewise conics. ACM Trans. on Graphics, 1989, 8(2): 112—121.